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Original article

Knowledge of the teaching staff and its relationship with the mathematical performance of the student body

Conocimiento del profesorado y su relación con el rendimiento matemático del estudiantado

Conhecimento do corpo docente e sua relação com o desempenho matemático do corpo discente

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ABSTRACT
Studies that estimate the statistical relationship between teacher knowledge and students' mathematical performance are limited and even more scarce in Latin America. From this point of view, the study aimed to examine the association between the teacher's knowledge and the student's performance, in the mathematical topic of fractions. Through a quantitative methodology, a non-experimental design of correlational and cross-sectional scope was used. Structured tests with closed questions were applied to 553 fourth grade students and 18 teachers from 18 Chilean schools. The test administered to teachers measures conceptual knowledge of fractions and knowledge about teaching. The study also examines the contextual variables: level of mathematical knowledge reported in national tests and socioeconomic level. Data is analyzed using Pearson correlations. The results show a positive, strong and significant correlation between conceptual knowledge and knowledge about teaching ($r = 0.71$ $p < .01$). The correlation between the teacher's conceptual knowledge and the student's performance is positive, weak but significant ($r = 0.31$ $p < .01$). The correlation between knowledge about teaching and student performance is positive, weak but significant ($r = 0.27$ $p < .01$). These results are in line with studies that suggest that teachers' knowledge plays an important role in student performance in mathematics.

Keywords: primary school teacher; mathematics teaching; learning; math; education.

RESUMEN
Los estudios que estiman la relación estadística entre el conocimiento del profesor y el rendimiento matemático de los alumnos son limitados y aún más escasos en Latinoamérica. Desde este punto de vista, el estudio tuvo como objetivo examinar la asociación entre el conocimiento del profesor y el rendimiento del estudiante, en el tema matemático de las fracciones. Mediante una metodología cuantitativa, se utilizó un diseño no experimental de alcance correlacional y transversal. Se aplicaron pruebas estructuradas con preguntas cerradas a 553 estudiantes de cuarto grado y a 18 profesores de 18 escuelas chilenas. La prueba administrada a los profesores mide el conocimiento conceptual de las fracciones y el conocimiento sobre la enseñanza. En el

estudio también se examinan las variables contextuales: nivel de conocimientos matemáticos reportados en pruebas nacionales y nivel socioeconómico. Los datos se analizan utilizando correlaciones de Pearson. Los resultados muestran una correlación positiva, fuerte y significativa entre el conocimiento conceptual y el conocimiento sobre la enseñanza (r= 0,71 p < ,01). La correlación entre el conocimiento conceptual del profesor y el rendimiento del estudiantado es positiva, débil pero significativa (r= 0,31 p < ,01). La correlación entre el conocimiento sobre la enseñanza y el rendimiento del estudiantado es positiva, débil pero significativa (r= 0,27 p < ,01). Estos resultados están en línea con estudios que sugieren que el conocimiento de los docentes juega un papel importante en el desempeño de los estudiantes en matemáticas.

Palabras claves: profesor de primaria; enseñanza de las matemáticas; aprendizaje; matemáticas; educación.

RESUMO

Estudos que estimam a relação estatística entre o conhecimento do professor e o desempenho matemático dos alunos são limitados e ainda mais escassos na América Latina. Sob esse ponto de vista, o estudo teve como objetivo examinar a associação entre o conhecimento do professor e o desempenho do aluno, no tópico matemático de frações. Por meio de uma metodologia quantitativa, foi utilizado um delineamento não experimental de abrangência correlacional e transversal. Testes estruturados com perguntas fechadas foram aplicados a 553 alunos da quarta série e 18 professores de 18 escolas chilenas. O teste aplicado aos professores mede o conhecimento conceitual de frações e o conhecimento sobre o ensino. O estudo também examina as variáveis contextuais: nível de conhecimento matemático relatado em provas nacionais e nível socioeconômico. Os dados são analisados usando correlações de Pearson. Os resultados mostram uma correlação positiva, forte e significativa entre conhecimento conceitual e conhecimento sobre ensino (r= 0,71 p < ,01). A correlação entre o conhecimento conceitual do professor e o desempenho do aluno é positiva, fraca mas significativa (r= 0,31 p < .01). A correlação entre o conhecimento sobre o ensino e o desempenho dos alunos é positiva, fraca mas significativa (r= 0,27 p < ,01). Esses resultados estão de acordo com estudos que sugerem que o conhecimento dos professores desempenha um papel importante no desempenho dos alunos em matemática.

Palavras-chave: professor primário; ensino de matemática; aprendizagem; matemática; educação.

INTRODUCTION

During the last three decades, attempts to better conceptualize and measure teachers' professional knowledge and thereby examine its relationship with student mathematical performance have increased. Studying teacher knowledge is a complex task, given that it has theoretically been modeled as a multidimensional construct. However, while some research shows that it does have several dimensions, others indicate that it is a one-dimensional construct. Thus, the distinction between the teacher's mathematical knowledge and the components related to teaching ability provides mixed findings. Consequently, more evidence is required to understand the nature of the teacher's knowledge (Copur-Gencturk & Tolar, 2022).

Shulman (1986) was a pioneer in proposing the following dimensions: Content Knowledge (CC), Pedagogical Content Knowledge (CPC) and curricular knowledge.

The CC is a central component, it refers to the teacher's knowledge regarding the content to be taught and its organizational structure. CPC relates to teaching ability, including knowing how to represent content in a way that others understand. It also covers student misconceptions and understanding why certain topics may be easy or difficult for them to learn. The curriculum is represented by a variety of materials available in programs designed for the teaching of the subject.

Shulman's (1986) work captured the attention of researchers and led them to further specify CC and CPC. In particular, in the field of educational mathematics, various components of the knowledge required to teach have been described. For example, Ball et al. (2008) presented the Mathematical Knowledge for Teaching (MKT) model made up of CC subdomains: common knowledge of the content, specialized knowledge of the content, and knowledge of the mathematical horizon. In addition, they established three CPC subdomains: knowledge of students and mathematical content, knowledge of teaching content, and knowledge of the curriculum. Currently, the Ball et al. (2008) has been widely cited and has become an important international reference.

In order to provide scientific evidence, researchers have carried out quantitative studies, estimating the association between teachers' CC and CPC with student performance (Cueto et al., 2017; Tchoshanov et al., 2017). However, in Latin America these studies are scarce, and even more on specific mathematical content, such as fractions.

Knowledge of fractions is associated with performance in mathematics. This theme has been reported to predict algebra learning and is consequently useful for learning more advanced mathematical concepts (Stelzer et al., 2021). However, worldwide many students experience difficulties understanding fractions (Xu et al., 2022). On the other hand, the literature suggests that not only students have difficulties in understanding fractions, but also teachers in service (Pouta et al., 2021). This understanding is critical given that teachers' knowledge of the mathematics they teach has an impact on the quality of instruction (Copur-Gencturk, 2021).

Several factors have been identified that contribute to explain the difficulty in learning fractions. However, most researchers agree that an important factor is that the fraction concept does not comprise a single construct but several sub-constructs. Kieren (1976) was the first to separate the concept of a fraction into five subconstructs: part-whole, operator, quotient, ratio, and measure. For example, 3/5 can be thought of as a part-whole (three out of five equal parts of a whole), as an operator (three fifths of a quantity), as a quotient (three divided by five), as a ratio (three is a five) and as a measure (a sum of three units of measure 1/5). It is precisely the understanding of these different meanings, one of the factors that contributes to explain the difficulty of their learning.

Based on the work of Kieren (1976), researchers have examined the difficulties that students present in the conceptualization of fractions. In general, studies reveal that students obtain better results in tasks related to part-whole and develop little knowledge in the other subconstructs (Fokides & Alatzas, 2022). Research shows that understanding the measured subconstruct turns out to be more difficult for students (Jiang et al., 2020).

On the other hand, there are studies that reveal that teachers show a limited understanding of fractions and that they present similar difficulties to students (Copur-Gencturk, 2021). To successfully address the difficulties faced by students, teachers must know and understand in depth the mathematics they are teaching, this would contribute to improve teaching. Consequently, it is pertinent to study the teacher's knowledge of fractions and their teaching and their relationship with the performance of fourth grade students.

For the purpose of this work, the conceptual knowledge of fractions is part of the teacher's CC, which involves understanding the subconstructs part-whole, quotient, operator, and measure (Kieren, 1976). Knowledge about teaching fractions is part of the teacher's CPC and is defined as didactic knowledge related to students' typical errors, difficulties, and strategies used in problem solving (Ball et al., 2008). The study also analyzes the variables: Socioeconomic Level (SEL) and the level of knowledge that schools reach in the mathematics tests of the Education Quality Measurement System (SIMCE).

Historically, in Chile there has been a gap between the mathematical academic achievement of students according to their SEL, which is permanently evidenced in national and international evaluations (del Río et al., 2022). Indeed, it is known that the NSE is related to the mathematical performance of students and that this is different for each school. The NSE is also related to the dependency of the establishment (municipal, private subsidized). Students with a higher SEL score significantly higher on the SIMCE national math test than those with a lower SEL.

Based on the exposed background, the study aimed to examine the association between teachers' knowledge and student performance in fractions. The following specific objectives are proposed:

1) Determine the relationship between the variable's knowledge of the teacher (includes conceptual knowledge and teaching), conceptual knowledge of fractions and knowledge about teaching them.

2) Determine the relationship between student performance in fractions, conceptual knowledge of teachers and knowledge about teaching.

3) Determine the relationship between student performance in fractions, the SIMCE and the NSE.

4) Explore the difficulties presented by students and teachers in fractions.

MATERIALS AND METHODS

The research was developed from a quantitative approach with a non-experimental design of correlational and cross-sectional scope. The study instruments were structured tests with closed questions applied to the students and their respective teachers. The analysis included two teacher variables: conceptual knowledge of fractions and knowledge about teaching fractions. Two contextual variables SIMCE and NSE. A student variable: knowledge of fractions.

The study involved 553 fourth grade students and 18 teachers from 18 schools, 7 municipal and 11 privates subsidized, which corresponds to 20% of the establishments in the city of La Serena, Chile. The groups were selected by proportional sampling with voluntary participation. The data was taken during the 2015-2016 school year (see Table 1). The teachers have the title of basic education teacher, which means that they teach classes in all subjects and mathematics. Of the total number of teachers, 15 are women and 3 are men, and on average they have 13 years of experience, with a minimum of 2 years and a maximum of 42 years.

Table 1- Types of school and students by school

schools	Guy	students
E01	P Sub	56
E02	mun	25
E03	mun	19
E04	P Sub	11
E05	mun	33
E06	P Sub	16
E07	P Sub	41
E08	P Sub	17
E09	mun	7
E10	P Sub	28
E11	P Sub	48
E12	P Sub	47
E13	P Sub	58
E14	mun	18
E15	mun	9
E16	mun	26
E17	P Sub	76
E18	P Sub	18
Total	18	553

Instruments

For data collection, a test of closed questions was used, which was applied to 553 fourth grade students once the teachers finished the fractions unit. The questions were organized according to a matrix of specifications of the "Fractions" unit framed in the curricular contents. The test was validated by expert judges in the topic of didactics of fractions. The instrument showed a reliability of 0.77 according to Cronbach 's alpha coefficient. Two tests were used for teachers, one of them on the conceptual knowledge of fractions framed in the curricular contents and another on the teaching of fractions based on the literature review. The test was validated by expert judges in the topic of didactics of fractions. Both tests showed a reliability of 0.75 according to Cronbach's alpha coefficient.

Context variables

SIMCE: Score obtained by the school in the annual SIMCE test carried out in the country, which evaluates the learning achievement in the subject of mathematics and covers a representative sample of the contents that must be treated in third and fourth grade. The scores of the SIMCE mathematics test are available online. NSE: It is measured through the economic income of the families and the educational level reached by the parents of the students who annually take the SIMCE test in the country. Two groups are considered in the study: Low SES and Medium High SES. In the Low SEL group, the parents declared having up to 10 years of schooling and an income of up to $340,000. In the medium-High SES group, the parents declared having between 13 and 15 years of schooling and an income between $550,001 and 1,250,000. NSE scores are available online. The high SES group whose parents declared having more than 15 years of schooling and an income of more than $1,250,000 did not participate in the study. Approximately 10% of the total establishments in the city of La Serena, Chile corresponds to this group.

Procedures

In order to apply the tests to the students, authorization was requested from the teachers, directors, attorneys-in-fact. The tests were applied by a project research assistant, in the first hours of classes, giving 60 minutes of time. The test was administered to the teachers in a room, giving 70 minutes of time to answer the questions of the test "Conceptual knowledge of fractions" and the questions of the test "Knowledge about the teaching of fractions".

Data is analyzed using Pearson correlations.

RESULTS

The correlations between the variable's teacher knowledge (C_Teacher_F), conceptual knowledge about fractions (Conceptual_F) and knowledge about the teaching of fractions (Teaching_F) were studied. The variable C_Profesor_F includes Conceptual_F and Enseñanza_F. All the correlations between these variables were positive, high and significant (see Table 2).

Table 2- Correlations between the teacher variables

No. = 553	Conceptual_F	Teaching_F
C_Teacher_F	0.92(**)	0.93(**)
Conceptual_F		0.71(**)

Note: Bilateral Pearson correlation; ** p < .01

Subsequently, the correlations between the variable student performance in fractions (Test F) and the variables C_Professor_F, Conceptual_F, and Teaching_F were studied. All the correlations between the Test F variable and the variables related to the teacher's knowledge were positive, weak but significant (see Table 3).

Table 3- Correlations between the student-teacher variables

No. = 553	C_Teacher_F	Conceptual_F	Teaching_F
F -test	0.31(**)	0.31(**)	0.27(**)

Note: Bilateral Pearson correlation; ** p < .01

Finally, the correlations between the variable Test F, the NSE, SIMCE were studied. It was observed that the highest correlation occurred between NSE and SIMCE. The correlation between the SIMCE variable and Test F was positive, moderate and significant. The association between performance on Test F and SIMCE (r= 0.50 p< 0.01) evidences the convergent validity of the test scales on fractions for students and the SIMCE test (see Table 4).

Table 4- Correlations between the school-student variables

No. = 553	SIMCE	F -test
NSE	0.60(**)	0.27 (**)
SIMCE		0.50(**)

Note: Bilateral Pearson correlation; ** p < .01

Test scores

Test on fractions applied to fourth grade students (n=553). Most of the students correctly answered the questions referring to the part-whole subconstruct (>80%), the questions that were difficult were those related to the measure subconstruct (20% to 50%), such as comparing fractions with different denominators and locating fractions on the number line.

Test on fractions and their teaching applied to teachers (n=18). Regarding the conceptual knowledge dimension about fractions, the questions that were easier for teachers to answer correctly were those referring to the part-whole subconstruct and the questions that were more difficult were those related to the measure subconstruct. Regarding the dimension of knowledge about teaching, the questions that were easier to answer correctly were those related to identifying strategies and common difficulties of the student body, and the questions that were more difficult were those related to the knowledge of common errors of the student body.

DISCUSSION

In the work it is observed that the correlation between the conceptual knowledge of the teacher and the knowledge about the teaching of fractions is positive, strong and significant (r= 0.71 p< .01; Table 2). This result is in line with research that shows that the mathematical knowledge of primary school teachers is strongly correlated with components related to teaching (Copur-Gencturk & Tolar, 2022). Some research has suggested that primary school teacher knowledge is a one-dimensional construct. However, it is currently not clear that mathematical knowledge is separable from teaching, so more evidence is required to understand the nature of the teacher's professional knowledge (Copur-Gencturk & Tolar, 2022).

In the study it was evidenced that the correlation between the conceptual knowledge of teachers and student performance in fractions is positive, weak but significant (r= 0.31 p < .01; Table 3). The correlation between knowledge about teaching and student performance in fractions is positive, weak but significant (r= 0.27 p < .01; Table 3). These results are similar to research that reports a significant association between teachers' knowledge and student performance in mathematics (Cueto et al., 2017; Tchoshanov et al., 2017). For example, the results of the study by Tchoshanov et al. (2017) show a statistically significant correlation (r= 0.29 p < .01) between the teacher's mathematical knowledge and student performance. Cueto et al. (2017) found a weak, positive but significant correlation between teachers' pedagogical content knowledge and student achievement in mathematics (r = 0.17 p < .05).

The study also confirmed a strong association between socioeconomic level and the average scores of the SIMCE mathematics test (r= 0.60 p < .01; Table 4). In addition, both the NSE and the SIMCE are significantly associated with student performance on the fractions test (r=0.27 and r=0.50 respectively; Table 4). This result is in line with what is reported in the literature. There is a significant association between the SES and the academic achievement in mathematics of Chilean students, which is permanently evidenced in national SIMCE and international assessments such as TIMSS and PISA (del Rio et al., 2022).

Regarding the results of the tests applied, it was evidenced that the questions that were easier for the students to answer were those related to the part-whole subconstruct and the most difficult were those related to the measurement subconstruct, such as locating fractions on the number line. and compare fractions with unlike denominators.

Some research indicates that students obtain better results in tasks related to part-whole and develop little knowledge in the other subconstructs (Fokides & Alatzas, 2022). Understanding the measured subconstruct turns out to be difficult for students (Jiang et al., 2020).

In general, in school textbooks simple models are used to represent part-whole. For example, geometric figures or images of cakes, chocolates or pizzas divided equally. The researchers point out that traditional teaching focuses on fractions as part-whole, being the understanding of this subconstruct the most dominated by students, paying little attention to the other subconstructs (Fokides and Alatzas, 2022). Consequently, typical errors appear: students err in stating that 1/3 > 1/2, arguing that 3 is greater than 2. While part-whole forms the basis for understanding the other subconstructs, this is not enough to have a deep understanding of fractions.

On the other hand, the measure subconstruct is represented by number lines, this model is less intuitive and therefore more difficult for students to understand (Jiang et al., 2020). In the same way, it is observed that the most difficult questions for the teachers to answer were those related to the measurement subconstruct. Some research indicates that the learning of fractions can be limited by the understanding that teachers have regarding the subject, if so, teacher educators have a key role in solving the problem (Copur-Gencturk, 2021). It is suggested that teacher educators reinforce knowledge of fractions and pay attention not only to the use of models that represent part-whole but also to the use of other models such as the number line.

The study adds to a body of research examining the relationship between teachers' knowledge and student achievement in mathematics. The work focused on the conceptualization of fractions, a topic that constitutes the basis for understanding more advanced mathematics (Stelzer et al., 2021). Therefore, identifying the types of teacher knowledge that are most strongly related to performance in fractions turns out to be of great interest for the field of educational mathematics, an area of research to which the results of this study open up. Consequently, the study constitutes a contribution to the investigation that examines this relationship. It is suggested that in the future, study the relationship in other populations of teachers and include other variables, such as attitudinal factors of teachers towards student learning.

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The author participated in the design and writing of the work, and analysis of the documents.

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