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Methodology for the development of logical-mathematical thinking from the demonstration by complete induction

Metodología para el desarrollo del pensamiento lógico-matemático desde la demostración por inducción completa

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ABSTRACT
This article is the result of research on logical procedures associated with the application of the mathematical induction demonstration method to contents related to numerical successions; the study is based on insufficiencies in the teaching-learning process of the subject Mathematics in the twelfth grade, which have their basis in theoretical inconsistencies and the tendency to the mechanical execution of procedures without logical arguments. In order to contribute to solving this problem, the methods used were bibliographic review, pedagogical test and systemic-structural; mathematical The main result obtained is a methodology that includes four models of mathematical activities with their teaching procedures; it could be concluded that an adequate conception of mathematical activities, consistent with the theory of numerical successions and the use of heuristic resources for inductive reasoning enhance logical-mathematical thought in the demonstration by mathematical induction.

Key words: Logical-mathematical thought; mathematical induction; numerical successions; method of demonstration.

RESUMEN
Este artículo es el resultado de una investigación sobre procedimientos lógicos asociados a la aplicación del método de demostración por inducción matemática a contenidos relacionados con las sucesiones numéricas; el estudio se realiza a partir de insuficiencias en el proceso de enseñanza-aprendizaje de la asignatura Matemática en el duodécimo grado, que tienen su base en inconsistencias teóricas y la tendencia a la ejecución mecánica de procedimientos sin argumentos lógicos; para contribuir a resolver esta problemática se emplearon los métodos revisión bibliográfica, la prueba pedagógica y el sistémico-estructural; el principal resultado obtenido es una metodología que comprende cuatro modelos de actividades matemáticas con sus procedimientos de enseñanza; se pudo concluir que una adecuada concepción de las actividades matemáticas, consecuente con la teoría de

sucesiones numéricas y la utilización de recursos heurísticos para el razonamiento por inducción, potencian el pensamiento lógico-matemático en la demostración por inducción matemática.

Palabras clave: pensamiento lógico-matemático; inducción matemática; sucesiones numéricas; método de demostración.

INTRODUCTION

One of the main objectives of the teaching of Mathematics is to develop in students a logical, flexible and creative thinking. Rational thinking is the object of study of Psychology and Logic; this manifests itself as a cognitive psychic process and as a result.

According to Petrovski (1985) , thought can be classified according to the content of the object that generates it, in that sense it recognizes thought: figurative, practical, logical and scientific; of the last two it should be noted that there is no one without the other, but in what way one thinks scientifically without taking into account the laws of logic.

For the author mentioned above, the thought is classified as logical because it follows the laws of Logic, so when this thought is developed in the field of Mathematics, we must speak of a thought, by logical nature, for the field of mathematics, that is, a logical-mathematical thought.

In Parada (2014) the need for the teacher to develop in students a mathematical logical thinking in order that these find most useful ways of representing the content through analogies, illustrations, examples, explanations and demonstrations is reaffirmed.

In addition, after making an analysis of several definitions given by experts and researchers, Herlina (2015) agrees that characterizes logical-mathematical thinking as «the cognitive process that includes representation, abstraction, creativity and mathematical demonstration» (p.2). Then these processes require conscious attention from the teaching-learning process. In essence, it is considered that enhancing the ability to demonstrate and in particular the demonstration by mathematical induction constitutes an indispensable way for the development of this type of thinking.

Another approach from which the issue of the development of logical-mathematical thinking is approached is that referred to the influence of students' beliefs about Mathematics (Diego, 2019, p.69). These determine the development of competences and the way in which students acquire prominence in the development of their intellectual capacities from the teaching-learning process of Mathematics.

According to Valenzuela (2018) "... the strategies for the generalization of patterns also constitute a way for the development of thought" (p.51). These are based on the study of a pattern through the sequential analysis of figures or mathematical objects to determine a general expression that allows calculating the value of any term of the sequence. This phenomenon manifests itself very regularly in the study of numerical successions and the demonstration of its general properties from mathematical induction constitutes a form of generalization.

Within the contents of the program of the Mathematical subject for the twelfth grade in the Cuban upper secondary education is the demonstration method by complete induction, this is introduced with the objective of equipping the student with tools that allow the demonstration of certain Mathematical properties. This method is used to demonstrate properties related to numerical sequences, divisibility problems, geometric problems, game theory, among others. The ability to demonstrate properties in the domain of natural numbers using the method of demonstration by full induction is one of the main objectives to be evaluated in systematic evaluations, partial control work and final examination of said subject.

Currently, it is the tendency of teachers to weigh the use of the method in the field of successions and numerical series, perhaps because the procedures are simpler, relating the activities to the demonstration that from certain general formulas is possible to calculate the sum of the first terms of a given numerical sequence. These contents have a theoretical basis that is subsequently studied in depth in the Mathematical Analysis in Higher Education courses.

During the teaching practice, the spaces for the preparation of the Mathematics subject and in the debates that take place in the courses of overcoming teachers, it has been possible to verify limitations of methodological order regarding the teaching of the demonstration method by complete induction and The associated concepts. These are manifested in theoretical inconsistencies when addressing issues of successions and numerical series. In addition, the teaching of the method, as teaching content, is mechanistic and does not follow the logic of mathematical inductive reasoning, resulting in the impossibility of applying it to multiple problems.

Cognitive insufficiencies have been verified as regularity year after year, in the systematic, partial and final evaluations. These are revealed in the mechanical application, without a logical basis, of the method of demonstration by mathematical induction. The deficiencies in the methodologies used by teachers, where there are limitations in the teaching of the method as content in itself, do not provide the student with the necessary tools to argue their reasoning when facing the solution of tasks with different levels of demand.

Another aspect that influences these inadequacies is that, the way in which the contents are presented to students in classes and assessments, contrasts with the activities presented in the twelfth grade Mathematics textbook authored by Campistrous, (1999).

Due to the need to deepen the causes that underlie the aforementioned insufficiencies, it is considered necessary to carry out a study on the concepts related to the process of development of logical-mathematical thinking from the demonstration by mathematical induction.

Taking into account the need to promote didactic experiences that contribute to the use of work with the method of mathematical induction as a resource to develop the logical-mathematical thinking of pre-university students, this article aims to publicize a methodology that consists of four activity models in which the contents of numerical sequences and the respective procedures for the demonstration process are coherently articulated.

MATERIALS AND METHODS

In order to obtain the methodology shown in this article, an applied research was developed, whose main basis was the use of the systemic-structural method, this focused on four fundamental aspects: The formalization with which the content is presented to the students (which becomes four activity models); work with concepts prior to the demonstration; the logic of the demonstration process and the forms of application associated with the property shown.

The design of the methodology was based on a bibliographic review of logical-mathematical thinking and general theoretical-methodological bases for its development; we proceeded to look for the theoretical support that appears in the books of higher mathematics referred to numerical sequences and Psychopedagogics references related to induction reasoning.

Subsequently, previous studies were conducted aimed at knowing the limitations of the students in the reasoning followed during the demonstration by mathematical induction, as well as the methodological limitations of the professors and the documents that regulate their work, which included a pedagogical test, a group interview to professors and the revision of the documents that regulate the work of the professor.

The pedagogical test was aimed at diagnosing three fundamental aspects: knowledge of the concepts of numerical succession, termination of a succession and succession of partial sums; the logic of the hypothetical reasoning in the step of the veracity of particular cases to the hypothesis and thesis approach as well as the level of skill development demonstrated by the method of mathematical induction in properties of arithmetic sequences.

Its application was based on an intentional sampling, for this the Pre-University Vocational Institute of Exact Sciences (IPVCE) «Federico Engels» of the province of Pinar del Río was selected. This institution was chosen because it is a study center where students have a better preparation compared to the other Pre-University students of the province. Once the school was selected, the best students were chosen from the ten twelfth groups including the contest students, in total 32 students.

In the application of the pedagogical test, we proceeded in such a way that the evaluator participated as a moderator based on the development of a group of indications and questions that allowed him to guide the students in obtaining the answers, while expressing the fundamentals of their reasoning, which allowed to know how they thought about the development of each one of the actions of the demonstrations carried out.

In the methodology followed by the evaluator for the search of the information in the exchange with the students, the following actions were taken:

1. Check if the students were able to identify the difference between what is a term of the succession of partial sums and term of the succession of addends.

2. Check if the students can identify the validity of the proposal to be demonstrated for {n " N: ne"2}.

3. Verify the presence of a hypothetical thought by generalizing, based on the

veracity of particular cases, that the proposition is valid for n = k.

4. Verify if students are able to perform conversion functions within the semiotic representation register, in this case algebraic. This aspect included the reasoning for the development of the passage from the hypothesis to the thesis.

5. Determine if students are able to verify the reasoning process developed during the demonstration (verify the equality between the succession of partial sums and the general term of a numerical series).

To process the information, a tabulation of the results was first made by indicators, the evaluation of the results was conditional, that is, what the student developed by himself was taken into account, contrasted with the answers he gave to the evaluator's questions. The results were analyzed in percentage calculations.

In the case of teachers, a group debate was held with the following aspects:

The debate began with the questioning of questions aimed at inquiring about theoretical knowledge about the method of demonstration by mathematical induction (the methodological resources used to teach the demonstration method around the logical step from the beginning to the hypothesis and from this, to the thesis and its demonstration) and the concepts associated with the theory of numerical successions and series, as well as the influence of these concepts on the internal aspect of the method (the way the student visualizes the structure of the entire demonstration process and its influence on the development of logical-mathematical thinking).

Subsequently, the methodological documents that support the work of the Professor of Mathematics in the Pre-University referred to the teaching of the demonstration method by complete induction were reviewed and contrasted with the ideas that emerged in the debate with the professors during the preparation of the subject.

The cognitive insufficiencies manifested by the students, the methodological limitations presented by the professors and the need for a theoretical foundation in the textbooks led to the development of a methodology for the development of logical-mathematical thinking, from the demonstration by mathematical induction.

For the implementation of the methodology, the preparation of the professors was developed during the course of overcoming, in which 33 mathematics teachers from all over the province of Pinar del Río participated.

In a first topic, the fundamental concepts and definitions of the theory of sequences and numerical series as well as the terminology to be used during the design of the activities were addressed. During the preparation, concepts of numerical successions were discussed and how the questions asked of the students should be formulated with the proper formalization, so that there were no contradictions with the theory referring to the successions and numerical series.

The second topic addressed the logical structure of the theorem: the principle of mathematical induction and its demonstration, the procedure to follow during the demonstration process according to the indications described in the methodology and the logical foundations of the beginning, hypothesis, thesis and demonstration.

RESULTS

During the investigation process it was obtained as results that:

94% of students have limitations in the development of an inductive-hypothetical reasoning, which is manifested in the deficiencies to identify the difference between the concepts: term of the succession of partial sums and term of the succession of addends. The essence of this difficulty lies in the inability to visualize the essential characteristics of these concepts, given by the symbols of addition between the terms.

In relation to the previous result, it was also found that 88% of the students fail to demonstrate the validity of the proposals for a value greater than the initial value.

91.2% of the students act mechanically in the hypothesis approach said mechanical reasoning is due to the contradiction manifested in the impossibility of verifying the truthfulness of the propositions for a value greater than the initial one, however they do not visualize that when posing the hypothesis for this brings it implicitly.

61.8% of students have limitations to perform treatment functions within the registry of semiotic representation, in this case algebraic. These difficulties are manifested in the students' inability to visualize equivalent expressions of the same concept, given by formalization in algebraic language.

Regarding the methodological limitations of the teachers, during the debates in the preparation sessions, it was found that there are inadequacies in the conceptualization of the contents related to the formalization and differentiation of the concepts: numerical succession and succession of partial sums. The tendency to guide the mechanical execution of the induction start was also observed without bearing in mind that only the concepts coincide for the case where $n = 1$, or for the first element with which it is checked.

It was also found that teachers guide the automated passage from hypothesis to thesis through the following sentence: "To obtain the thesis you just have to substitute in equality, n by k + 1", but it is a purely mechanical and memoristic action, which breaks with all logical foundation of the reasoning.

It was found that there are no methodological orientations in the program of the twelfth grade Mathematics subject for the treatment to the demonstration by complete induction, in this it refers to the book: Methodology of Mathematics teaching, Volume I (Ballester, 2007, p. 328), where a brief general presentation of the demonstration method is carried out as a tool that allows to demonstrate properties in the domain of natural numbers. In this book, the general method is described in broad strokes but does not go deeper into the logical foundation necessary for the start-hypothesis-thesis and demonstration steps. In addition, it is necessary to analyze the application of the method in the context of the sequences.

With the objective of eradicating the methodological limitations that teachers have in this subject and the cognitive insufficiencies of the students, a methodology is proposed in order to develop in students a logical-mathematical thought from the demonstration by mathematical induction.

Methodology for the development of logical-mathematical thinking from the demonstration by complete induction

Based on the concept of methodology proposed by De Armas & Valle (2011), the methodology is understood as a sequence of actions and orientations that indicate the step-by-step instructions by teachers to develop logical-mathematical thinking from work with the method of demonstration by mathematical induction. The actions to be developed and the guidelines for each are described below.

1. Identify the type of activity model to develop.

2. Treatment of the fundamental concepts of successions and numerical series.

3. Treatment of the fundamental elements of the method of demonstration by mathematical induction.

4. Development of tasks associated with the method of demonstration and evaluation of the transformations achieved in the student.

According to the proposed methodology, it is considered that, for students to succeed in using the method of demonstration by mathematical induction and the associated concepts, the first thing that should be known by the teacher is how to formulate the statements and questions, of so that there are no contradictions with the theory of successions.

Four models for mathematical activities are discussed below and the methodological procedure for each case is argued:

1. Let the sum be 8 + 12 + 16 + ... + (4n + 4) = S_n, ne"1

2. Let 8 + 12 + 16 + ... + (4n + 4) be the general term of the sequence {S_n}, ne"1

3. Let the arithmetic sequence be {A_n} = {8; 12; 16;…; (4n + 4);…}; ne"1 and {S_n} the succession of partial sums.

4. Let the function be a: N '! R such that a (n) = 4n + 4 for all ne"1 and S_n = a (1) + a (2) + a (3) + ... + (4n + 4)

In the previous activities the notation of successions has been taken according to (Valdés & Sánchez, 2017). In the first case, there is no reference to the term "succession", so it is subsequently not recommended to include the term in the subordinate subsections, and if so, it should be clarified to whom reference is made, to the succession of addends or the succession of partial sums. In the second case, reference is made to the succession of partial sums, so if one were to ask about any term of the succession of addends, it should be left explicit.

In the third case, the arithmetic succession and the succession of partial sums of said succession are made explicit, although the nth term of {S_n} is not raised . In this case, the difference between the terms of the sequence of addends and the terms of the sequence of partial sums is better visualized. In the fourth case, succession in functional language is discussed as a particular case. In this the concepts of succession terms acquire the name of functional values, sum of the images of the function, or sum of functional values.

In any of the cases, if one chooses to state an activity, it is necessary to be consistent with the theory of successions either in the language of "term of" or "values of the function." Such a choice should not give rise to confusion if the reference is to arithmetic succession or to the succession of partial sums. Now let's see how to correctly ask the questions for each example:

A. It is requested to calculate a sum given the position *k*

- In example 1: "Say the sum of S_n that is in position *k".*

- In Example 2: "Say the sum of the general term of the sequence S_n that is in the position *k* ".

- In example 3: "Say the term k of the sequence $\{A_n\}$", or "say the term of the sequence $\{A_n\}$ in position k".

- In example 4: "Calculate a (k)", or "determine the value of the function for n = k".

For this type of question, the student must evaluate n = k (the position given) in the expression generated by the addends, that is, in the general term of the arithmetic sequence. Special attention should be paid to the domain of definition of said sequence, if it is for ne"0, for ne"1 which can also be written as n> 0, or for n≥a. In any case, the following must be kept in mind:

- If the sequence is defined for ne"0 then there is an offset between the position of the requested term and the natural value by which n is substituted. This would be [(*n-ésima posición*)-1]. (Table 1)

Table 1 - Sequence Table

Position	one	two	3	... n
Substitution (n)	0	one	two	... (n-1)

In learning this case, it is recommended that the student rehearse the search for the first two or three terms so that he can visualize the difference between the position and the value he replaces.

- If the sequence is defined for n≥a, in this case the given position coincides with the natural value by which it is necessary to substitute to obtain the requested term.

- If the sequence is defined for ne"a, then there is an offset between the position of the term and the natural value by which n is substituted. In this case we would have to replace with [(n-ésima posición)+(a-1)]. Table 2 shows the sequence.

Table 2 - Sequence Table

Position	one	two	3	... n
Substitution (n)	to	a + 1	a + 2	... To + (n-1)

B. If you want to determine the position given an element of the arithmetic sequence.

- In example 1: a_i is a sum of S_n. Determine itsposition.

- In example 2: a_i is a sum of the general term of the sequence $\{S_n\}$. Determineits position.

- In example 3: a_i is a term of the sequence $\{A_n\}$. Determine its position, or is it a_i a term of the sequence $\{A_n\}$? Argue.

- In Example 4: Determine if there is, the natural value for which a_i is the image of the function.

In this type of question the student is proposed a term of arithmetic succession and is asked to determine the position. The procedure is for the student to match the general term of the sequence to the term from which the position is to be determined. For this situation the student must reason as follows "If $_i$ is a term of the sequence $\{A_n\}$ then there exists a natural number n for which $A_n = a_i$", the question would then find that n, which It is obtained by solving the equation $A_n = a_i$. In this case the domain of definition of the sequence must be kept in mind $\{A_n\}$, that is, if it is for n≥0, for n≥1 or for n≥a. Let's look at the three cases:

- If the sequence $\{A_n\}$ is defined for n≥0. Then to determine the position of the term a_i must add 1 to the solution of the equation, that is, if the solution of the equation is n the given term is in the position i = n + 1.

In order for the student to understand this type of reasoning, it is important that he first determine the position of the first two or three terms and compare the solution of the equation with the position of the terms.

- If the sequence $\{A_n\}$ is defined for n≥1, then the requested position of the term given to $_i$ coincides with the solution of the equation $A_n = a_i$, in this case i = n.

- If the sequence $\{A_n\}$ is defined for n≥a, then to determine the position of a_i the phase offset to the solution of the equation $A_n = a_i$ must be subtracted, that is, if n is the solution then i = n- (a-1), the following table 3 illustrates the procedure for the sequence {6n} when n≥3.

Table 3 - Sequence Table

Term to i	18	24	30	... 6a
Solution of A n = a_i	n = 3	n = 4	n = 5	... n = a
Position p i	p 1 = 1	p 2 = 2	p 3 = 3	... P i = i = n- (a-1)

C. Prove by the method of mathematical induction that the sum raised can be calculated by a certain formula S_n.

- In example 1: Prove by applying the principle of mathematical induction that $S_n = 2n (n + 3)$ for n≥1.

- In Example 2: It demonstrates by applying the principle of mathematical induction that the general term of $\{S_n\}$ is $S_n = 2n (n + 3)$ for n≥1.

- In Example 3: "Prove by applying the principle of mathematical induction that the sum of the first n terms of the arithmetic sequence $\{A_n\}$ is $S_n = 2n (n + 3)$ for n≥1, or "demonstrate by applying the principle of mathematical induction that the succession of partial sums $\{S_n\}$

has as a general term $S_n = 2n (n + 3)$".

- In Example 4: It demonstrates by applying the principle of mathematical induction that the sum of the first n functional values is calculated by the expression $S_n = 2n (n + 3)$.

Let us now analyze the procedure to develop the method of demonstration by mathematical induction. A very easy variant for the student to visualize the relationships of the property to be demonstrated is to write it as an equality between the expression that contains the sum of the terms of the arithmetic sequence and the expression that generates the partial sums. For example:

I. 8 + 12 + 16 + ... + (4n + 4) = 2n (n + 3) for n≥1 or it could also be written:

II. " $^{n}{}_{(i = 1)}$ (4i + 4) = 2n (n + 3)

Let's start the demonstration with the start of induction for n = 1. In this case it is necessary to observe that the important thing is to verify that the expression of the right member of equality produces the sum of the first n terms of the arithmetic sequence {4n + 4} and that they only coincide in the first term. Then it should be:

MI (left member): S_1 = 8 and MD (right member): 2 '" 1 (1 + 3) = 8. Then the property is valid for n = 1.

Although it is not essential, if it is important for the student to know that the property is valid for other values n = 2; 3…. This encourages the student to know the meaning of the expression he is demonstrating. It is convenient not only to comply with the veracity of the proposition for an initial value, but also to make a brief exploration of the veracity of the proposition for other values greater than. This could perhaps warn us of a bad reasoning before continuing. In addition, the veracity of some cases suggests the approach of a hypothesis.

For example, let's check that equality is also valid for n = 2 and n = 3 and respectively.

MI: S_2 = 8 + 12 = 20 MD: 2 '" 2 (2 + 3) = 4 '" 5 = 20. Then it is fulfilled.

MI: S_3 = 8 + 12 + 16 = 36 MD: 2 '" 3 (3 + 3) = 6 '" 6 = 36. Then it is fulfilled.

In this way the student is logically prepared to feel the need to formulate a hypothesis or a conjecture, not yet proven. From the veracity of particular cases the veracity of the proposition for an n = k is induced; it is assumed that the sum of the first k terms of {4n + 4} or addends of the left member of equality is true.

Induction hypothesis: The proposition is true for n = k. In this case the student must understand that this means that the sum of the first k addends can be calculated by the proposed expression for S_{n}. For example:

a) 8 + 12 + 16 + ... + (4k + 4) = 2k (k + 3)

b) S_k = 8 + 12 + 16 + ... + (4k + 4)

c) S_k = 2k (k + 3)

d) " $^{k}{}_{(i = 1)}$ (4i + 4) = 2k (k + 3)

All these expressions have the same meaning. The most used in Cuban secondary education is that of subsection a), this is obtained from an immediate

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combination of subsections b) and c). Now the reasoning would be the following "if for n = k the proposition is true, then for n = k + 1 it is also true."

Induction thesis: The proposition is also true for n = k + 1. It is necessary to emphasize the meaning of this step and relate it logically to the previous one.

One of the logical mistakes made by the student is to mechanically replace the position k by k + 1 in part a) above, being as follows: 8 + 12 + 16 + ... + (4k + 8) = 2 (k + 1) (k + 4). It is not bad to do it, the difficulty is in doing it and not being aware that now the last summing shown in the left limb has a k + 1 position. In this sense it is important to recognize that the previous sum is precisely (4k + 4). But what happens many times is that it is not known, or the teacher wants to avoid the effort of the explanation and what guides is an automatic procedure of substitution. So, how to do it?

If the sum has already been raised in the hypothesis until the sum k, simply add the following sum to that sum, "keeping the previous sum." In one of the following ways:

a) 8 + 12 + 16 + ... + (4k + 4) + (4k + 8) = 2 (k + 1) (k + 4)

b) $S_{(k + 1)}$ = 8 + 12 + 16 + ... + (4k + 4) + (4k + 8)

c) S_k + (4k + 8) = 2 (k + 1) (k + 4)

d) $S_{(k + 1)}$ = S_k + (4k + 8)

e) $S_{(k + 1)}$ = 2 (k + 1) (k + 4)

f) " $^{k}_{(i = 1)}$ (4i + 4) + (4k + 8) = 2 (k + 1) (k + 4)

g) " $^{k}_{(i = 1)}$ (4i + 4) = 2 (k + 1) (k + 4)

Although all are valid and equivalent, it is considered that in these ways of writing the thesis, subsection a) is more suggestive to visualize logic of direct demonstration, that is, from the hypothesis to obtain the thesis. Indeed, as the true hypothesis has been assumed, then let's start with this:

8 + 12 + 16 +... + (4k + 4) = 2k (k + 3). What is desired is to obtain the thesis; we must guide the student to a visual and logical process of comparison between the left members of the hypothesis and the thesis.

Hypothesis: 8 + 12 + 16 + ... + (4k + 4) = 2k (k + 3)

Thesis: 8 + 12 + 16 + ... + (4k + 4) + (**4k** + **8**) = 2 (k + 1) (k + 4)

In this way the student can have visual support for their reasoning, just have to ask what is left of the left member of the hypothesis to get the left member of the thesis?, you can easily see that it is the sum (4k + 8), then it is enough to add this to both members of the hypothesis and verify, from operations, that the right member of the thesis can be obtained. It should be noted that at the end of the demonstration the equality of the two members of the thesis has been obtained, which is what we wanted to demonstrate.

D. Questions associated with the succession of partial sums.

- In example 1: "Calculate S_{100}" or "Calculate 8 + 12 + 16 + ... + 404"

- In Example 2: "Calculate the 100 term of {S_n}", "Calculate 8 + 12 + 16 + ... + 404", or "Calculate S_{100}"

- In Example 3: "Calculate the sum of the first 100 terms of {A $_n$}", "Calculate the 100 term of {S $_n$}", "Calculate 8 + 12 + 16 + ... + 404", or "Calculate S $_{100}$ "

- In example 4: "Calculate the sum of the first 100 values of the function", "Calculate S $_{100}$ " or "Calculate a (1) + a (2) + a (3) + ... + a (100).

In all the previous cases, the equivalence of the questions should be highlighted depending on the activity model given in the four examples. Some simple demonstration questions can also be incorporated using the term general term of the sequence {S $_n$}, for example:

a) Show that the sum up to a multiple of 3 is always divisible by 18.

b) Prove that the sum of an even number of addends is always divisible by 4.

In fact, for subsection a) any sum up to a multiple of three is obtained for n = 3k from where it is obtained that S $_{3k}$ = 2 '" 3k (3k + 3) = 18k (k + 1) which is divisible by 18 In part b) S $_{2k}$ = 2 '" 2k (2k + 3) = 4k (k + 1) which is divisible by 4.

To construct activities of those analyzed in this investigation, it is enough to choose any general formula of the type {an + b}, (ne"1) which represents an arithmetic sequence, then we obtain the first three terms and add them until we obtain an expression of the type a_1 + a_2 + a_3 + ... (an + b). Then we use the expression S $_n$ = na $_1$ + $\frac{n(n-1)}{2}$ (ne"1) to obtain the particular expression of the general term of the sequence {S $_n$} being as follows:

a $_1$ + a $_2$ + a $_3$ + ... (an + b) = S $_n$ (ne"1)

If you want to define enough to add both members of the previous equality, that is:

0B + a 0$_1$ + a $_2$ + a $_3$ +... (an + b) = S $_n$ + b, (ne"0)

It could also extend to the domain of negative integers if we multiply by -1 both members of either of the two previous equalities:

-a $_1$ -a $_2$ -a $_3$ -… - (an + b) = - S $_n$ (ne"1).

One of the manifestations of the effectiveness of the methodology can be seen in the results achieved by the twelfth grade students of the province of Pinar del Río in question (P2) referring to the demonstration by mathematical induction of the final exam (EF). As can be seen in Table 4, 94.8 % of the approved students reached more than 12 points out of the 20 possible and 86.2 % of those approved reached the maximum points in the question.

Table 4 - Summary of the results of the final twelfth grade Math exam.

12th	**(EF)**	**P2 (EF)**	**P2 (20 points)**
Enrollment	2688	2686	2686
Approved	2686	2546	2314
Percent	99.9%	94.8%	86.2%

Source: Report on the results of the final exam of the Provincial Directorate of Education, Pinar del Río. Course 2018-2019.

DISCUSSION

The previous results show that during the conceptualization process, a coherence between the meaning, the formalization and the way in which the contents referring to the successions and numerical series are presented to the students, is based on Distéfano (2019) who considers that "meaning is determined by identification, syntax and semantics" (p.149). These elements are essential for the development of logical-mathematical thinking, while formalization depends on a correct definition and the essential differences that emerge according to the level of abstraction when visualizing a succession of partial sums as a particular case of succession.

These aspects are addressed in the methodology proposed from which the professor is suggested to contextualize the method of demonstration by mathematical induction in the case of numerical sequences. In this sense, the student must organize his logical-mathematical thinking from the mathematical activity model he faces. These models depend on the way in which the activity is stated and on the formalization in which the concepts of successions and series are presented; either as a sum, as a general term of a sum or in functional language because a sequence is a function of N '! R.

It coincides with D'Amore (2015), in order to achieve a development of students' mathematical logical thinking "it is necessary to relate semi-cognitive processes closely (treatment and conversion functions)" (p.183). These aspects are related to the ability: demonstrate by mathematical induction, an aspect that constitutes a difficulty because the students fail to harmonize the concepts of sequences and series by the formalization in which the contents are presented.

The results show that there are limitations in the identification of terms of the succession of addends and the succession of partial sums, confusing these concepts. This inconsistency is due to the way in which the question is formulated, because the theory of succession and its formal language is not consistent. This situation influences the student to establish a hypothesis without having based their ideas on an inductive reasoning, that is, said methodological procedure leads to the student formulating a mechanical hypothesis without a logical basis.

It is necessary that before the use of the demonstration method, students should consider activities related to the manipulation of previous concepts that will support all the reasoning of the demonstration that will be carried out later. This previous manipulation is closely related to the ways in which cognitive obstacles are avoided, which according to D'Amore (2015) are related to the specific characteristics of the only possible instrument of their denotation: semiotic systems (mathematical formalization) and states that in Today this is one of the most widespread topics in the world, in the research field.

Another important finding is the high probability of mistakes that a student commits when trying to mechanically execute the induction demonstration procedures. To solve this problem, the teacher must develop an inductive thinking in the student, that is, educate the student to verify certain property for particular cases, from these he infers a hypothesis and thesis that is then demonstrated.

As a novel element for the management of the method it is suggested that at the start of induction it should be noted that for n = a (initial value) the concepts associated with succession of addends and succession of partial sums coincide, therefore it is of the utmost importance that do more truth checks to notice the difference between these concepts. Furthermore, this procedure enhances induction reasoning (a general rule is induced from the veracity of particular cases). This aspect strengthens the rationalization of the students' practical activity during inductive reasoning.

According to the above, it is necessary for the students to present the thesis keeping in mind the sum up to the term n = k (to visualize that in the hypothesis a proposition was considered as valid until n = k) and then add the sum that is in the position n = k + 1. Otherwise, he is performing a mechanical procedure of substituting n for k + 1 without taking into account the meaning, in this case the student performs a procedure that is fine, but lacks logical meaning in his reasoning.

Another important aspect is that the logic of the reasoning that leads to the hypothesis lies in the possibility of making k truthfulness checks, which implies that the proposition is valid for n = k and if it is valid for n = k then it will be valid for n = k + 1 that would be the thesis to prove. This is essential for a qualitatively superior development of logical-mathematical thinking to occur, while the logic of the reasoning followed is significant.

The results obtained with the implementation of the proposed methodology show progress significantly superior to previous courses. The students evidenced, in their answers, coherence in the conceptual basis of the successions and numerical series as well as a logical procedural development in the demonstration by the method of mathematical induction. The deficiencies detected were minimal and were located in knowledge related to algebraic work.

It is a reality that there are theoretical and practical limitations in the use of the method of demonstration by mathematical induction, these are manifested in the low levels of student learning in this subject. First, these limitations have their origin in the fundamentals underlying the contents in which the method is implemented, say the concepts of: succession, termination of a sequence and place it occupies, partial sum and succession of partial sums, concepts which are studied in depth in Mathematical Analysis.

The ability to demonstrate and in particular demonstrate by the method of mathematical induction is one of the mechanisms by which logical-mathematical thinking is developed since demonstration is a characteristic process of it. It is in the twelfth grade of the Cuban school where the student is initialized in this type of activity, precisely to equip him with an important mathematical tool and at the same time contribute to the development of his logical thinking.

The way in which teachers present such content is essential to achieve effective and productive intellectual development in students. Therefore it is important to know how the student reasons during the use of the method to design effective methodological actions; these must be based on the principle of mathematical induction, on the correct formalization of the theory of sequences and numerical series, as well as, in logical arguments that allow deductive-hypothetical reasoning during the demonstration process.

The development of logical-mathematical thinking is manifested in the structured and generalized visualization of the procedures involved in a given activity; in the level of abstraction of the students for the identification of subordinate and collateral concepts, in the capacity to base the inductive-hypothetical reasoning and in the possibility of using the demonstration method in different contexts and problems.

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